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On the Deficiency of 8–10 Day Galactic Cepheids

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The Galactic Cepheid period histogram has a strong dip between 8 and 10 days that has defied an explanation based on evolutionary and linear pulsation studies. We show here that this deficiency is caused by the instability of the *nonlinear* fundamental pulsation cycle in this period range. The strong metallicity dependence of this instability is consistent with the absence of a corresponding minimum in the Magellanic Cloud data. Our results also suggest that the Galactic Cepheids must have a large spread in metallicity.

It is a well known fact that the observed period distribution of the Galactic Cepheids has a pronounced minimum in the 8–10 d period range, as shown in Fig. 1 which displays the histogram for the period distribution of the Galactic Cepheids constructed from the Galactic Cepheid Database of Fernie et al. (1995). Since it is difficult to separate overtone and fundamental pulsators for the Galaxy above 5 d our histogram necessarily contains both the fundamental and the overtone Cepheids.

Becker, Iben & Tuggle (1977) combined the location and duration of Cepheid model crossings of the instability strip from evolutionary calculations with a birth-rate function to infer a theoretical period histogram. One of their conclusions was that a minimum in the distribution was not compatible with a standard birth-rate function, and could only be explained if an ad hoc two-component birth-rate function were adopted (cf. however Chiosi 1989). Their evolutionary computations were performed with the now superseded Los Alamos opacities which are now known to be considerably too weak. However, while the new opacities will produce a different period distribution it is difficult to see how they might cause a two-humped one.

In this Letter we show that the minimum in the period distribution of Cepheids is a result of the nonlinear dynamics associated with the fundamental pulsations of the Cepheids, and that it has therefore nothing to do with evolutionary calculations.

The proper mass–luminosity (ML) relations for the Cepheid model sequences would normally have to be obtained from evolution calculations. However, at the present time there is enough uncertainty and disagreement (Buchler et al. 1996, Beaulieu et al. 1997) to lead us to determine these ML relations differently. From the structure of the Fourier decomposition parameters ϕ_{21} and R_{21} for the Galaxy and for the Magellanic Clouds one infers that the 2:1 resonance occurs in the vicinity of a period of 10 d. Based on this fact we construct the ML relation as follows: For each metallicity Z we determine the mass M and luminosity L for which the P_0=10 d equilibrium-model is resonant viz. P_2/P_0=1/2 (bump Cepheid) with an effective temperature T_{eff} that lies ΔT=100 K degrees to the right of the fundamental blue edge. (Mathematically, for given composition parameters X and Z and a given ΔT we solve for $P_0(M, L, T_{eff})$=10, $P_2(M, L, T_{eff})$=5 with Teff = T_{BE}–ΔT where T_{BE} = $T_{BE}(M, L)$). From these anchor values (M, L) we then derive a ML relation with a slope chosen to be 3.56 that is close to what evolutionary calculations indicate.

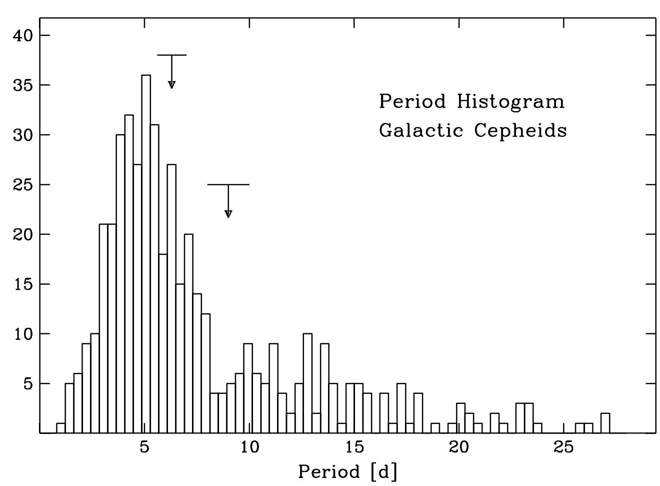

Fig.1: Period distribution of fundamental and overtone Galactic Cepheids d'après Fernie et al.

We use the Livermore OPAL95 opacities (Iglesias & Rogers 1996) combined with the molecular opacities of Alexander & Ferguson (1994), to compute the nonlinear fundamental pulsations with the relaxation method of Stellingwerf (1974). We put a temperature anchor at 11,000 K with 30 constant mass shells in the surface, and with 90 geometrically increasing zones inward. The pseudo-viscosity parameters are C_Q=4 and α=0.01. Convection is ignored, and we caution that the models lose their validity far from the blue edge. Concomitantly with the relaxation to the periodic pulsation the code performs a Floquet analysis of the limit cycles (i.e. the periodic finite amplitude pulsations) (e.g. Buchler 1990). We recall that the Floquet exponents measure the linear stability of the limit cycle to perturbations with respect to all the possible modes (e.g. Ince 1944): For a limit cycle to be stable, and thus to be observable, *all* Floquet exponents, λ_k, have to be negative. This stability analysis is an extremely useful byproduct of the relaxation method. In fact without the Floquet analysis we might have to integrate thousands of cycles to ascertain stability of a limit cycle, and, in the case of instability, we would not be able to compute the limit cycles at all and determine *how* unstable they are. The relaxation code on the other hand is robust enough to converge on a limit cycle even when the latter is mildly unstable.

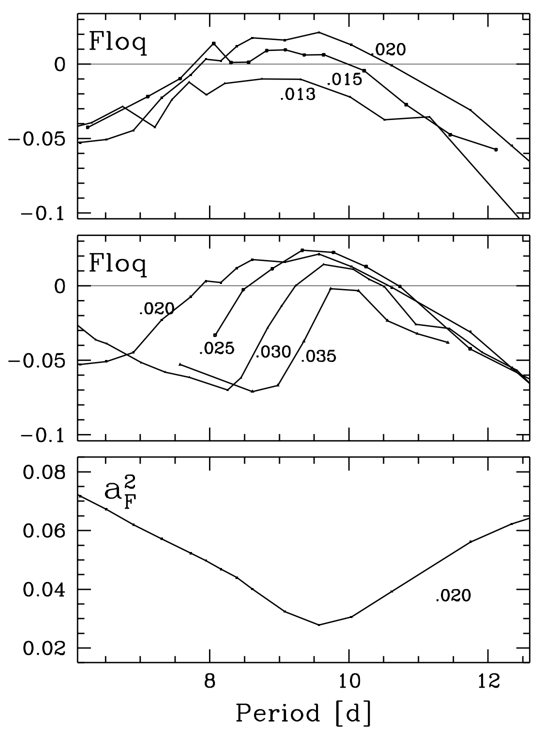

Fig.2: *Top, middle:* Floquet stability exponent λ_1 for the nonlinear fundamental Cepheid pulsations as a function of period and metallicity Z (with X=0.70) for models 100 K to the right of the fundamental blue edge; *Bottom:* Square of the *radius* Fourier amplitude of the limit cycle scaled by the period, a_F, cf text.

In Fig. 2 we exhibit the behavior of the Floquet stability exponent, λ_1, of the fundamental limit cycle as a function of its (nonlinear) period P_0^{nl}. (In this period region it is the perturbation with the first overtone that is the least stable). The six sequences have metallicities, with Z ranging from 0.013 to 0.035. In order to avoid cluttering we have split the figure into two subfigures, the top showing the metallicities Z= 0.013, 0.015 and 0.020, and the bottom the values 0.020, 0.025, 0.030 and 0.035. The hydrogen mass fraction has been chosen at X=0.70. All model sequences run ΔT=100 K to the right of the fundamental blue edge. For metallicities in the range Z $\approx$ 0.014 to 0.035 these fundamental limit cycles are thus unstable to a perturbation in the first overtone.

In order to see the effect of the location of the Cepheids with respect to the blue edge we display in Fig. 3 results obtained for a sequence with ΔT = 400 K (with the same ML relation as for the 100 K sequences described above). The width of the period region of unstable limit cycles thus depends somewhat on location with respect to the instability strip, and shrinks with ΔT.

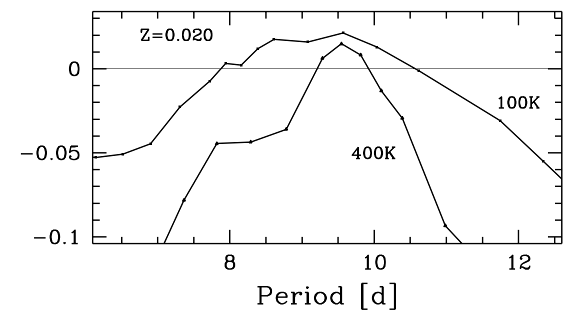

Fig.3: Floquet stability exponent λ_1 for the nonlinear fundamental Cepheid pulsations as a function of period for two sequences with ΔT = 100 and 400 K and with Z=0.020 and X=0.70.

Older nonlinear Cepheid pulsations that were performed with the now obsolete Los Alamos opacities (Moskalik & Buchler 1991) indicated that the fundamental Cepheid pulsations were stable in the vicinity of 10 d (except for an absolutely minute range, as their Fig. 1 shows). It is quite clearly the overall increase in the opacities that is responsible for the instability. One can get a good feeling for the sensitivity of the Floquet exponents to opacity from Fig. 3 of Buchler (1996) where the results of calculations with the OPAL95, the OPAL93 and the Los Alamos opacities are compared. Roughly speaking, the change from OPAL93 to OPAL95 is equivalent to an increase of Z from 0.02 to 0.03, for example (Buchler, Kolláth, Beaulieu & Goupil 1996).

We caution the reader though that there is some sensitivity of the Floquet exponents to the zoning and to other numerical parameters that are used in the nonlinear calculations (Kovács 1990, Yecko, Kolláth & Buchler 1997), as well as to convection whose effects we have ignored. The precise values of the metallicity Z for the onset of in-

stability and of the period range thereof should therefore not be taken too literally, although the *existence* of an instability in the broad 8–10 d range seems to be essentially independent of such numerical parameters.

What is the reason behind the instability of the fundamental limit cycles? We show now that the resonance between the fundamental mode of oscillation and the second overtone ($P_2/P_0 = 1/2$) cause an overall decrease of the pulsation amplitude that in turn destabilizes the fundamental limit cycle. We note that this is the same resonance that causes the familiar Hertzsprung bump progression.

It is a well known observational fact that the pulsation amplitudes of the classical Cepheids exhibit a substantial drop for periods in the vicinity of 10 d. The same feature is found in numerical modelling. The amplitude equation formalism (e.g. Buchler 1993) explains how nonlinear effects associated with the resonance P_2/P_0=1/2 between the fundamental and the second overtone are responsible for the drop in the overall pulsation amplitude. Furthermore, large amplitudes increase the stability of a limit cycle pulsation, as the same formalism shows. Indeed the Floquet exponent λ_1 behaves as (Buchler, Moskalik & Kovács 1991)

$$\lambda_1 = (\kappa_1 - q_{10}A_0^2 - q_{12}A_2^2)\, P_0 \tag{1}$$

where κ_1 is the linear growth-rate of mode 1, q_{1j} are nonlinear coupling coefficients that depend on the structure of the star, and A_j are the pulsation amplitudes of the excited modes, i.e. the fundamental and the resonant second overtone here.

For simplicity, let us make a few approximations. First we ignore the smaller amplitude of the second overtone, A_2 compared to A_0. Second, with good reasons we assume that the cubic coupling coefficients q are positive and that they scale as $q_{10} = c_{10}P_0$ (Kovács & Buchler 1989). Third we disregard the small variation of $\kappa_1 P_0$ over the plotted range of periods. In Eq. 1 the amplitudes are relative, i.e. they refer to $\delta R(t)/R$. Thus we associate A_0 with the lowest Fourier amplitude a_1 of the absolute computed radius variations, scaled by the stellar radius R_*, i.e. with $a_F = a_1/R_*$. How the radius scales with period can be found by the following rough estimate. Using the mass–luminosity relation $L \sim M^{3.56}$, the period–mean density relation $P_0 \sim \rho^{-1/2} \sim R^{3/2}M^{-1/2}$, the surface luminosity equation $L \sim R^2T^4$ and the shape of the blue-edge $T \sim L^\alpha$, with $\alpha \sim -0.05$, one finds

$$P_0 \sim R^\beta \quad , \qquad \beta = \frac{3}{2} - \frac{1}{3.56(1-4\alpha)} \sim 1.27$$

With $\beta \approx 1$ Eq. 1 is reduced to

$$\lambda_1 \approx \kappa_1\, P_0 - c_{10}\, a_F^2 \tag{2}$$

A comparison of the bottom panel of Fig. 2 with the upper panels clearly confirms the correlation between pulsation amplitude and stability.

On the basis of the stability properties of the fundamental limit cycles exhibited in Figs. 2 and 3 we thus reach the following conclusions. First, for metallicities in the range $Z \approx 0.014$ to 0.035 the fundamental limit cycle is unstable to a perturbation in the first overtone. Second, the window of instability shifts to higher period with increasing Z.

Turning now to the astronomical implications we note that the Galactic period histogram has a dip, but not an actual gap in the 8–10 d range as it should have if the metallicity of all Cepheids were as large Z=0.02. If the minimum of the histogram is now interpreted as a dispersion in metallicity the observed $\approx$ 8–10 d fundamental Cepheids, approximately a third, must therefore have lower metallicity, $Z < 0.013$ (or else an unlikely $Z > 0.035$). We note though that Becker et al. (1977), albeit on totally different grounds, also suggested that Galactic metallicity dispersion is large (a range Z_{max}/Z_{min} of 3 to 5).

However, it may be objected that since the stability of the fundamental limit cycles depends on the location with respect to the instability strip a wide strip could also reduce the Cepheid deficiency especially on the lower period side. There are evolutionary arguments (Becker et al. 1977) that the Cepheid instability strip may be much narrower than generally assumed. An estimation of the instability strip from the Fourier decomposition parameters also suggested a narrower rather than larger instability strip (Buchler, Moskalik & Kovács 1990).

How much of the survival of 8–10 d Cepheids is indeed due to a dispersion in metallicity rather than due to the width of the instability strip can be tested with an observational measurement of the metallicities of the individual Cepheids.

While there is a deficiency of fundamental Cepheid pulsators in the 8–10 d range, there is absolutely no reason to believe that there is a deficiency of the corresponding stars. As expected, hydrodynamical calculations show that these stars pulsate in stable first overtone limit cycles. They should therefore show up as an excess in the histogram at lower periods (lower by a factor ≈ 0.7, the typical period ratio P_1/P_0). Indeed, the histogram of Fig. 1 is certainly compatible with an excess in the 5.6–7.0 d range. (The left overbar shows the first overtone period range corresponding to the fundamental range shown on the right.)

Turning now to other galaxies, we recall that Becker et al. (1977) present also a period histogram for M31, a galaxy that also has a relatively high metallicity. Again the histogram shows the dip in the 8–10 d range, consistently with our results.

In contrast, the Magellanic Clouds are known to have considerably lower average metallicities than the Galaxy, and indeed, in agreement with our stability analysis, the period histograms for the Magellanic Clouds (Becker et al 1977) do not show much indication of a minimum.

The small hollow near 9 d, if statistically significant, may again be an indication of a spread in metallicity putting some Magellanic Cepheids above the threshold value of $Z \approx 0.013$.

If the 2:1 resonance at 10 d plays such an important role one may wonder if other resonances might make themselves felt, especially because the Galactic Cepheid period distribution (Fig. 1) seems to indicate a deficiency of Cepheids in two other places.

First, if the gap near $P \approx 25$ d is indeed significant it has an interesting implication. Numerical hydrodynamic calculations (Moskalik & Buchler 1991 and Moskalik, Buchler & Marom 1992 for the new opacities) show that the $P_0/P_1 = 3/2$ resonance can destabilize the fundamental Cepheid pulsation and lead to a *periodic* pulsation with double period, in which alternating cycles differ only slightly however. We note though that Fernie (preprint) has not found any evidence of alternations in his Cepheid data. However, in support of this scenario Antonello & Morelli (1996) has suggested that the star CC Lyr does exhibit alternations. The observational data of CC Lyr are very limited though, and additional observations of this star are necessary to confirm the existence of alternations. Is it possible that the dip in Fig. 1 could arise because such stars with alternating cycles may not have been classified as Cepheids? If this resonance is indeed strong enough to cause instability to alternations then this would add an additional observational constraint for Cepheid models.

Second, Fig. 1 perhaps also suggests a minimum in the vicinity of 19 d. One notes that this is the period region where the fundamental mode is in a 4:1 resonance with the fifth overtone (P_5/P_0=1/4). However, the linear stability of the 4th overtone is quite large, and a hydrodynamical survey is necessary to verify whether it can play a sufficiently large dynamical role to destabilize the fundamental limit cycle.

In conclusion, the simple and natural explanation for the deficiency of $\approx$8–10 d Cepheid variables is that the fundamental mode limit cycles are unstable. The corresponding stars pulsate in the first overtone with period $P_1 \approx 0.7 P_0$, giving rise to a relative excess of overtone Cepheids in the corresponding range of $\approx$5.6–7.0 d periods. It is thus not necessary to invoke an ad hoc two-component birth-rate function to explain the Cepheid period distribution.

Our calculations and the agreement they provide with the minimum in the observed Galactic Cepheid distribution, and the lack of a minimum in the lower metallicity Large Magellanic Cloud data also provide a further confirmation that the new opacities are in the right ball-park. Indeed, if they were much weaker, the 8–10 d fundamental Cepheid pulsations would all be stable, thus not giving the observed minimum. On the other hand if they were much stronger, then they would predict a minimum for the LMC as well, and a smaller one, or none at all for the Galactic Cepheids, in contrast with observation.

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